Name ________________________ ID3 _ _ _

ECS 315: Probability and Random Processes **2017/1**

HW 8 — Due: Oct 31, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

Let A_1 be the event that the first part is classified correctly

(a) This assignment has 4 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

A_2 " - - - - second - - - - -

(d) (8 pt) Try to solve all problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that three parts are inspected and that the classifications are independent.

(a) Let the random variable X denote the number of parts that are correctly classified. Determine the probability mass function of X. [Montgomery and Runger, 2010, Q3-20]

Possible values of X are $0, 1, 2, 3$

$P(A_1 \cap A_2 \cap A_3^c) + P(A_1 \cap A_2^c \cap A_3) + P(A_1^c \cap A_2 \cap A_3)$

$P[X=2] = \binom{3}{2}(0.98)^2(0.02)$

$P[X=k] = \binom{3}{k}(0.98)^k(0.02)^{3-k}$ $\Big\}$ $k = 0, 1, 2, 3$

$$p_X(x) = \begin{cases} \binom{3}{x}(0.98)^x(0.02)^{3-x}, & x \in \{0,1,2,3\}, \ x = 0, 1, 2, 3, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Let the random variable Y denote the number of parts that are incorrectly classified. Determine the probability mass function of Y.

Possible values of Y are $0, 1, 2, 3$

$P[Y=k] = \binom{3}{k}(0.02)^k(0.98)^{3-k}$, $k = 0, 1, 2, 3$

$p_Y(y) = \Big\{$

Problem 2. Consider the sample space $\Omega = \{-2, -1, 0, 1, 2, 3, 4\}$. Suppose that $P(A) = |A|/|\Omega|$ for any event $A \subset \Omega$. Define the random variable $X(\omega) = \omega^2$. Find the probability mass function of X.

ω	$X(\omega) = \omega^2$
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16

Possible values of X are $0, 1, 4, 9, 16$

$$P[X=0] = P(\{\overset{\omega}{0}\}) = \frac{|\{0\}|}{|\Omega|} = \frac{1}{7}$$

$$P[X=1] \equiv P([X=1]) \equiv P(\{\omega : X(\omega) = 1\})$$

$$= P(\{1, -1\}) = \frac{2}{7}$$

ω

$p_X(x) = \Big\{$

Problem 3. Suppose X is a random variable whose pmf at $x = 0, 1, 2, 3, 4$ is given by $p_X(x) = \frac{2x+1}{25}$.

Remark: Note that the statement above does not specify the value of the $p_X(x)$ at the value of x that is not 0,1,2,3, or 4.

(a) What is $p_X(5)$?

(b) Determine the following probabilities:

(i) $P[X = 4]$

(ii) $P[X \leq 1]$

(iii) $P[2 \leq X < 4]$

(iv) $P[X > -10]$

Problem 4. The random variable V has pmf

$$p_V(v) = \begin{cases} cv^2, & v = 1,2,3,4, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} \frac{1}{30}v^2, & v = 1,2,3,4, \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of the constant c.

$c = \frac{1}{30}$

$\{1, 4, 9, 16, 25, \ldots\}$

$\boxed{P[V \in B] \qquad B = \{1, 4, 9, 16, 25, \ldots\}}$

(b) Find $P[V \in \{u^2 : u = 1,2,3,\ldots\}]$.

$= P[V = 1] + P[V = 4] = p_V(1) + p_V(4) = \frac{1}{30}(1^2 + 4^2) = \frac{17}{30}$

(c) Find the probability that V is an even number.

(d) Find $P[V > 2]$.

(e) Sketch $p_V(v)$.

(f) Sketch $F_V(v)$. (Note that $F_V(v) = P[V \leq v]$.)

Problem 5. The thickness of the wood paneling (in inches) that a customer orders is a random variable with the following cdf:

$$F_X(x) = \begin{cases} 0, & x < \frac{1}{8}, \\ 0.2, & \frac{1}{8} \le x < \frac{1}{4}, \\ 0.9, & \frac{1}{4} \le x < \frac{3}{8}, \\ 1 & x \ge \frac{3}{8}. \end{cases}$$

Determine the following probabilities:

(a) $P[X \le 1/18]$

(b) $P[X \le 1/4]$

(c) $P[X \le 5/16]$

(d) $P[X > 1/4]$

(e) $P[X \le 1/2]$

[Montgomery and Runger, 2010, Q3-42]